

Fermi National Accelerator Laboratory

FERMILAB-Conf-97/266-T

Realizing the Potential of Quarkonium

Chris Quigg

*Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510*

July 1997

Presented at the *Twenty Beautiful Years of Bottom Physics*,
IIT, Chicago, Illinois, June 29-July 2, 1997

Disclaimer

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, expressed or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Distribution

Approved for public release; further dissemination unlimited.

Realizing the Potential of Quarkonium

Chris Quigg ¹

*Fermi National Accelerator Laboratory ²
P.O. Box 500, Batavia, Illinois 60510 USA*

Abstract. I recall the development of quarkonium quantum mechanics after the discovery of Υ . I emphasize the empirical approach to determining the force between quarks from the properties of $c\bar{c}$ and $b\bar{b}$ bound states. I review the application of scaling laws, semiclassical methods, theorems and near-theorems, and inverse-scattering techniques. I look forward to the next quarkonium spectroscopy in the B_c system.

PROLOGUE

I am very happy to share in this celebration of the twentieth anniversary of the discovery of the b -quark. The upsilon years were a very special time for the development of particle physics. Reviewing the events of two decades ago, I was struck not only by the pace of discovery, but by how easy it was, and how much fun we had. As a young physicist of that time, I am grateful not only for the excellent science, but also for the excellent people that quarkonium quantum mechanics gave me an opportunity to know, work with, and even compete with. They taught me a great deal about physics and life.

THE EMPIRICAL APPROACH

Charmonium quantum mechanics was already well-launched when the Υ came along. Appelquist & Politzer [1] had shown that nonrelativistic quantum mechanics should apply to $Q\bar{Q}$ systems. The Cornell group [2] had shown the predictive power of the nonrelativistic potential-model approach using a “culturally determined” potential,

¹) Internet address: quigg@fnal.gov.

²) Fermilab is operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the United States Department of Energy.

$$V(r) = -\frac{\kappa}{r} + \frac{r}{a^2} . \quad (1)$$

Eichten & Gottfried [3] had anticipated the spectroscopy of $b\bar{b}$, predicting

$$\begin{aligned} M(\Upsilon') - M(\Upsilon) &\approx 420 \text{ MeV} \\ &\approx \frac{2}{3}[M(\psi') - M(\psi)] . \end{aligned} \quad (2)$$

All this meant that the deductive approach—assuming a form for the interquark potential and calculating the consequences—was in very good hands.

Jon Rosner and I were motivated to take a complementary empirical approach by the way the quarkonium problem came into our consciousness. Shortly after the discovery of Υ , Taiji Yamanouchi visited us to ask, “What does it mean that the $\Upsilon' - \Upsilon$ spacing is the same as $\psi' - \psi$? Could the potential be in between Coulomb and linear—like a logarithm?”

The facts we had at our disposal in the summer of 1977 were these:

E288	$M(\Upsilon') - M(\Upsilon)$	$M(\Upsilon'') - M(\Upsilon)$
Two-level fit	$650 \pm 30 \text{ MeV}$	
Three-level fit	$610 \pm 40 \text{ MeV}$	$1000 \pm 120 \text{ MeV}$
$M(\psi') - M(\psi)$	$\approx 590 \text{ MeV}$	

We found [4] that for a power-law potential of the form $V(r) = \lambda r^\nu$, level spacings scale as

$$\Delta E \propto (2\mu/\hbar^2)^{-\nu/(2+\nu)} |\lambda|^{2/(2+\nu)} , \quad (3)$$

where μ is the reduced mass. Note that for the Coulomb problem, the scaling law (3) leads to the familiar result, $\Delta E \sim \mu|\lambda|^2$ ($m\alpha^2$). Further, we found that a logarithmic potential, $V(r) = C \ln r$, corresponds to the limit of a power-law as $\nu \rightarrow 0$, so that

$$\Delta E \sim C\mu^0 , \quad (4)$$

a level spacing independent of μ .

The logarithmic potential gives a good account of the (then known) ψ and Υ spectra, as shown in Figure 1, but is not unique in doing so. It was an easy matter to produce a modified Coulomb + linear potential that gave equal spacing for the ψ and Υ families, but for no other quark masses. Now, the logarithmic potential is the solution to an idealized statement of the experimental facts. We expect it to be a good representation of the interaction in the region of space that governs the properties of the narrow ψ and Υ states, but we have no reason to attach fundamental significance to it. It is mildly intriguing that a logarithmic confining interaction emerges from the light-front QCD approach when the Hamiltonian is computed to second order [5].

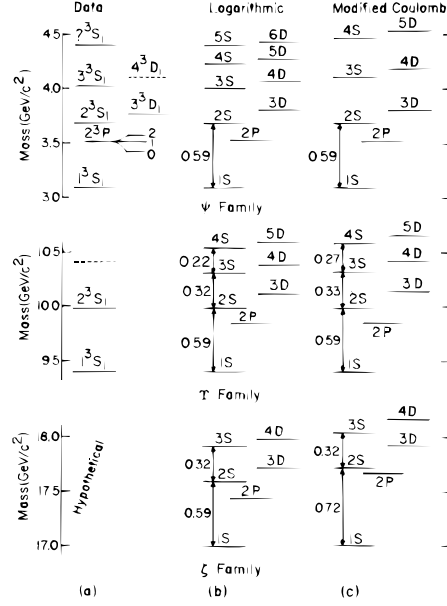


FIGURE 1. Level schemes of the ψ , Υ , and hypothetical ζ families in (a) nature, (b) the logarithmic potential, and (c) a Coulomb + linear potential. (From Quigg & Rosner [4].)

SCALING THE SCHRÖDINGER EQUATION

The Schrödinger equation for the reduced radial wavefunction in a potential $V(r) = \lambda r^\nu$ is

$$\frac{\hbar^2}{2\mu} u'' + \left[E - \lambda r^\nu - \frac{\ell(\ell+1)\hbar^2}{2\mu r^2} \right] u(r) = 0 . \quad (5)$$

With the substitutions

$$r = \left(\frac{\hbar^2}{2\mu|\lambda|} \right)^{1/(2+\nu)} \rho , \quad (6)$$

$$E = \left(\frac{\hbar^2}{2\mu|\lambda|} \right)^{-2/(2+\nu)} \left(\frac{\hbar^2}{2\mu} \right) \varepsilon , \quad (7)$$

and the identification $w(\rho) \equiv u(r)$, we can bring the Schrödinger equation to dimensionless form [6],

$$w''(\rho) + \left[\varepsilon - \text{sgn}(\lambda)\rho^\nu - \frac{\ell(\ell+1)}{\rho^2} \right] w(\rho) = 0 . \quad (8)$$

The substitution (6) means that quantities with dimension of length scale as $[L] \propto (\mu|\lambda|)^{-1/(2+\nu)}$, whereas (7) tells us that quantities with dimensions of energy scale as $[\Delta E] \propto (\mu)^{-\nu/(2+\nu)} (|\lambda|)^{2/(2+\nu)}$. The scaling behavior in several familiar potentials is shown in Table 1.

TABLE 1. How length and energy observables scale with coupling strength $|\lambda|$ and reduced mass μ in power-law and logarithmic potentials.

Potential	$[L]$	$[E]$
Coulomb	$(\mu \lambda)^{-1}$	$\mu \lambda ^2$
Log: $V(r) = C \log r$	$(C\mu)^{-1/2}$	$C\mu^0$
Linear	$(\mu \lambda)^{-1/3}$	$\mu^{-1/3} \lambda ^{2/3}$
SHO	$(\mu \lambda)^{-1/4}$	$\mu^{-1/2} \lambda ^{1/2}$
Square well	$(\mu \lambda)^0$	μ^{-1}

Quarkonium Decays

The scaling laws have immediate applications to the matrix elements that govern quarkonium decays. Electric and magnetic multipole matrix elements have dimensions

$$\langle n' | E_j | n \rangle \sim L^j, \quad (9)$$

$$\langle n' | M_j | n \rangle \sim L^j / \mu. \quad (10)$$

Radiative widths are given by

$$\Gamma(E_j \text{ or } M_j) \sim p_\gamma^{2j+1} |\langle n' | E_j \text{ or } M_j | n \rangle|^2, \quad (11)$$

so that transition rates scale with mass as

$$\Gamma(E1) \sim \mu^{-(2+3\nu)/(2+\nu)}, \quad (12)$$

$$\Gamma(M1) \sim \mu^{-(4+5\nu)/(2+\nu)}. \quad (13)$$

Probability densities have dimensions L^{-3} . Accordingly, the wave function squared at the origin scales as

$$|\Psi(0)|^2 \sim \mu^{3/(2+\nu)}, \quad (14)$$

so the leptonic width of a vector meson scales as

$$\begin{aligned} \Gamma(\mathcal{V}^0 \rightarrow \ell^+ \ell^-) &= 16\pi\alpha^2 e_Q^2 |\Psi(0)|^2 / M(\mathcal{V}^0)^2 \\ &\sim \mu^{-(1+2\nu)/(2+\nu)}. \end{aligned} \quad (15)$$

Combining (12) and (13) with (15), we see that

$$\Gamma(E1) / \Gamma(\ell^+ \ell^-) \sim \mu^{-(1+\nu)/(2+\nu)}, \quad (16)$$

$$\Gamma(M1) / \Gamma(\ell^+ \ell^-) \sim \mu^{-3(1+\nu)/(2+\nu)}. \quad (17)$$

For a potential less singular than a Coulomb potential, radiative decays become relatively less important than leptonic decays, as μ increases.

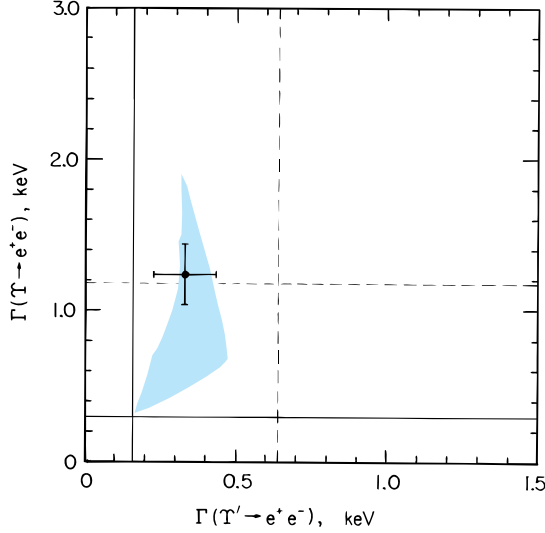


FIGURE 2. Expectations for the leptonic widths of Υ and Υ' . The lower bounds (19) are indicated for $e_b = -\frac{1}{3}$ (solid lines) and $e_b = \frac{2}{3}$ (dashed lines). The shaded region shows the widths predicted for $e_b = -\frac{1}{3}$ on the basis of twenty potentials from [20] that reproduce the ψ and ψ' positions and leptonic widths. The data point represents the 1978 DORIS results.

Measuring the b -quark's charge

For a power-law potential with $\nu \leq 1$, the scaling law (14) implies that

$$|\Psi_b(0)|^2 \geq \frac{m_b}{m_c} |\Psi_c(0)|^2, \quad (18)$$

which leads to

$$\Gamma(\Upsilon_n \rightarrow \ell^+ \ell^-) \geq \frac{e_b^2}{e_c^2} \cdot \frac{m_b}{m_c} \cdot \frac{M(\psi_n)^2}{M(\Upsilon_n)^2} \Gamma(\psi_n \rightarrow \ell^+ \ell^-). \quad (19)$$

The inequality holds for more general potentials than power laws. We can prove it for the ground state for any monotonically increasing potential that is concave downward [7]. For excited states we have given a derivation for the same class of potentials in semiclassical approximation.

The numerical bounds that follow from (19) are indicated in Figure 2. Measurements presented at the 1978 Tokyo Conference by the collaborations working at the DORIS storage ring [8],

$$\begin{aligned} \Gamma(\Upsilon \rightarrow \ell^+ \ell^-) &= 1.26 \pm 0.21 \text{ keV} , \\ \Gamma(\Upsilon' \rightarrow \ell^+ \ell^-) &= 0.36 \pm 0.09 \text{ keV} , \end{aligned}$$

ruled decisively in favor of the $e_b = -\frac{1}{3}$ assignment. The fifth quark was indeed *bottom*.

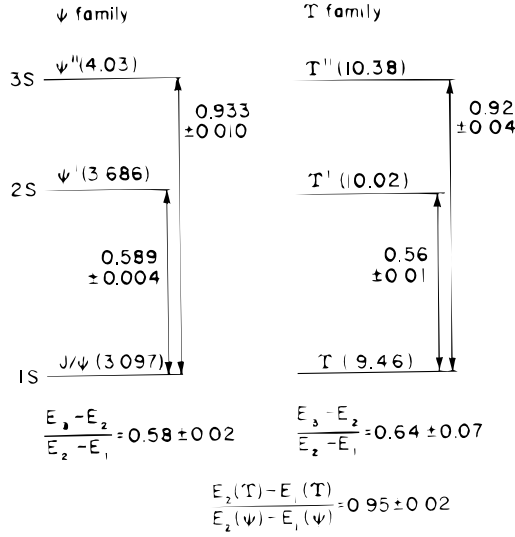


FIGURE 3. Comparison of 3S_1 levels of the ψ and Υ families, circa 1979.

The Order of Levels

The discovery in 1975 of the narrow resonances P_c and χ confirmed the theoretical expectation that the 2S and 2P levels are not degenerate in charmonium, as they would be in a pure Coulomb potential. In response to the question of what the 2S–2P splitting says about the interquark potential, André Martin and collaborators (R. Bertlmann, H. Grosse, J.-M. Richard, and others) constructed a series of elegant theorems on the order of levels in potentials [9,10].

After early data on the Upsilon confirmed the similarity of the J/ψ and Υ spectra, as shown in Figure 3, Martin used the scaling laws to deduce a simple power-law potential,

$$V(r) = -8.064 \text{ GeV} + (6.898 \text{ GeV})(r \cdot 1 \text{ GeV})^{0.1} . \quad (20)$$

The Martin potential [11] has served as a very useful template for the J/ψ , Υ , and even $\phi(s\bar{s})$ families. In addition, it has led to many informative predictions for the masses of baryons containing charm and beauty.

A Priority Dispute with Isaac Newton

About three years ago, Jon Rosner telephoned me to say that Professor Chandrasekhar had just advised him that we were in a priority dispute with Isaac Newton. “Capitulate at once!” I said. “Newton can be very terrible.”

In writing his superb commentary on the *Principia Mathematica* [12], Chandrasekhar had found that Newton was the first to explore pairs of dual power-law

potentials, and that he had mapped the Kepler problem into the harmonic oscillator. We had shown [13]—three centuries later—that the bound-state spectrum of an infinitely rising power-law potential,

$$V(r) = \lambda r^\nu \quad (\nu > 0) \quad , \quad (21)$$

is connected with that of a singular potential,

$$\bar{V}(r) = \bar{\lambda} r^{\bar{\nu}} \quad -2 < (\bar{\nu} < 0) \quad . \quad (22)$$

The paired Schrödinger equations for the two cases can be written as

$$\frac{\hbar^2}{2\mu} u''(r) + \left[E - \lambda r^\nu - \frac{\ell(\ell+1)\hbar^2}{2\mu r^2} \right] u(r) = 0 \quad , \quad (23)$$

$$\frac{\hbar^2}{2\mu} v''(z) + \left[\bar{E} - \bar{\lambda} z^{\bar{\nu}} - \frac{\bar{\ell}(\bar{\ell}+1)\hbar^2}{2\mu z^2} \right] v(z) = 0 \quad , \quad (24)$$

where $(\nu+2)(\bar{\nu}+2) = 4$ and $\bar{E} = \lambda(\bar{\nu}/\nu)^2$, $\bar{\lambda} = -E(\bar{\nu}/\nu)^2$, $(\bar{\ell}+1/2)^2\nu^2 = (\ell+1/2)^2\bar{\nu}^2$, and $z = r^{1+\nu/2}$.

The familiar quantum-mechanical correspondence between the Coulomb and harmonic oscillator problems emerges as a special case. For circular orbits, Newton cites a relation between ν and $\bar{\nu}$ equivalent to ours. We did capitulate, and so far we have not suffered any indignities at Newton's hands. And because Jon is a scholar, he and Aaron Grant have written an excellent historical review of the classical and quantum-mechanical analyses [14].

COUNTING NARROW LEVELS OF QUARKONIUM

Eichten & Gottfried had argued that a $Q\bar{Q}$ system with $m_Q \gg m_c$ would have at least three narrow 3S_1 levels [3]. As Kurt recalled in his talk, this implies a very rich spectroscopy, which figured prominently in the scientific case for CESR. However, the observed $\Upsilon' - \Upsilon$ spacing is much larger than the 420 MeV they predicted. Figure 4 shows that in the logarithmic potential, we would predict three or four narrow 3S_1 levels of Υ , in agreement with Eichten & Gottfried's expectation. This circumstance led us to ask how general is the expectation, and on what does it depend?

Semiclassical methods led us to a remarkable general result [15]: The number of narrow 3S_1 levels is

$$n \approx 2 \cdot \left(\frac{m_Q}{m_c} \right)^{1/2} . \quad (25)$$

The derivation is short enough to reproduce in full.

Set the zero of energy at $2m_Q$. The threshold for the dissociation of quarkonium $(Q\bar{Q}) \rightarrow Q\bar{q} + \bar{Q}q$ lies at an excitation energy $\delta(m_Q)$. If $V(r)$ binds $Q\bar{Q}$

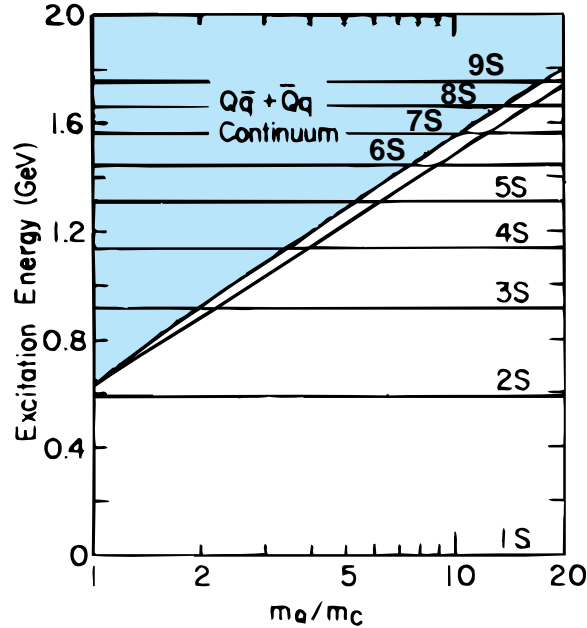


FIGURE 4. Plot of the $Q\bar{q} + \bar{Q}q$ threshold relative to the (ground-state) 1^3S_1 $Q\bar{Q}$ level as a function of the ratio of the heavy-quark mass m_Q to the charmed quark mass m_c for a logarithmic potential. Upper curve: reduced-mass and hyperfine corrections included. Lower curve (straight line): reduced-mass and hyperfine corrections ignored. Horizontal lines denote the $2^3S_1, 3^3S_1, \dots Q\bar{Q}$ levels in this potential. (From Quigg and Rosner [15].)

states rising at least $\delta(m_Q)$ above $2m_Q$, then the WKB quantization condition is

$$\int_0^{r_0} dr [m_Q(\delta(m_Q) - V(r))]^{1/2} \simeq (n - \frac{1}{4})\pi, \quad (26)$$

where r_0 is the point at which $V(r_0) = \delta(m_Q)$. As Eichten & Gottfried had observed, $\delta(m_Q) \equiv 2m(\text{lowest } Q\bar{q} \text{ state}) - 2m_Q$ approaches a finite limit δ_∞ , independent of m_Q , as $m_Q \rightarrow \infty$. This means that the only dependence of (26) on the heavy-quark mass is the explicit factor of $\sqrt{m_Q}$ on the left-hand side. We have, by inspection, the general result

$$(n - \frac{1}{4}) \propto \sqrt{m_Q}. \quad (27)$$

This universal behavior is realized in different ways for different potentials, as illustrated in Figure 5. The examples chosen are $V(r) = r$, $V(r) = \ln r$, and $V(r) = -r^{-1/2}$, for which $\Delta E \propto \mu^{-1/3}$, μ^0 , and $\mu^{+1/3}$, respectively. All the levels fall deeper into the potential as the reduced mass μ increases, in conformity with the Feynman–Hellmann theorem. For the singular potential with $\nu = -\frac{1}{2}$, the levels spread apart as they sink into the well. For the linear potential, the levels are packed more densely as μ rises. The logarithmic potential is an intermediate case in which the level spacing is independent of

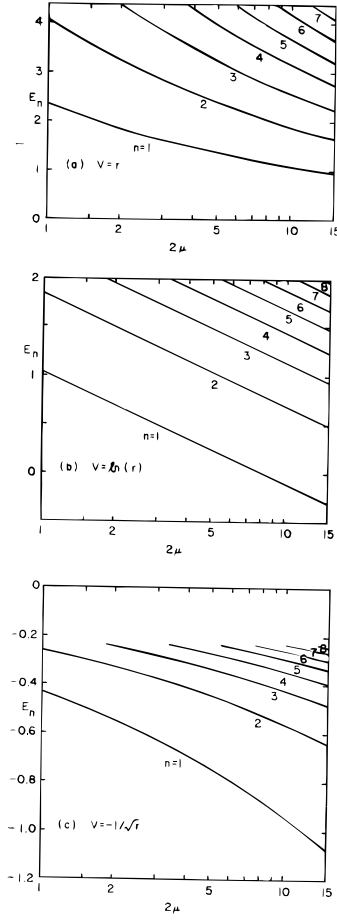


FIGURE 5. Comparison of the reduced-mass dependence of energy levels in three potentials: (a) $V(r) = r$; (b) $V(r) = \ln r$; (c) $V(r) = -r^{-1/2}$.

the mass and all levels fall into the well at a common rate given by $E_i(\mu') = E_i(\mu) - \frac{1}{2} \ln(\mu'/\mu)$.

SEMICLASSICAL METHODS AND RESULTS

The power of the WKB approximation for the counting problem encouraged us to explore other applications of semiclassical methods. Evaluating the nonrelativistic connection

$$|\Psi_n(0)|^2 = \frac{\mu}{2\pi} \left\langle \frac{dV}{dr} \right\rangle_n \quad (28)$$

in the semiclassical approximation, we connect the square of the s -wave wave function at the origin to the level density:

$$|\Psi_n(0)|^2 = \frac{(2\mu)^{3/2}}{4\pi^2} E_n^{1/2} \frac{dE_n}{dn} \quad (29)$$

(for a nonsingular potential) [16]. For example, in a linear potential $V(r) = \lambda r$, $|\Psi_n(0)|^2$ is independent of n , by (28), so $E_n \sim n^{2/3}$.

Using the connection

$$\Gamma_n \equiv \Gamma(\mathcal{V}_n^0 \rightarrow \ell^+ \ell^-) = 16\pi\alpha^2 e_Q^2 \frac{|\Psi_n(0)|^2}{M(\mathcal{V}_n^0)^2} , \quad (30)$$

we can derive a variety of semiclassical sum rules, including

$$\sum_{n=\text{narrow}} \frac{\Gamma_n}{M_n^p} \simeq \frac{4\alpha^2 e_Q^2 m_Q^{3/2}}{\pi} \int_0^\delta \frac{dE E^{1/2}}{(2m_Q + E)^{2+p}} , \quad (31)$$

where $\delta = 2M(Q\bar{q}) - 2m_Q$. These are useful in evaluating the heavy-quark mass m_Q , and in calculating the cross section for heavy-quark photoproduction using vector-meson-dominance techniques.

The connection (29) was generalized to higher partial waves by Bell and Pasupathy [17], who found

$$\left| \frac{d^\ell R_{n\ell}(r)}{dr^\ell} \right|_{r=0}^2 = \frac{1}{\pi} \left[\frac{\ell!}{(2\ell+1)!!} \right]^2 \left(\frac{2\mu E_{n\ell}}{\hbar^2} \right)^{\ell+1/2} \frac{\partial(2\mu E_{n\ell}/\hbar^2)}{\partial n} , \quad (32)$$

and generalized to include singular potentials by Moxhay & Rosner [18].

For a monotonically increasing potential, the semiclassical quantization condition

$$\int_0^{r_0} dr [2\mu(E - V(r))]^{1/2} = (n - \frac{1}{4})\pi \quad (33)$$

connects the shape of the potential to the level density:

$$r(V) = \frac{2}{2\mu^{1/2}} \int_0^V dE (V - E)^{1/2} \left[\frac{dE_n}{dn} \right]^{-1} . \quad (34)$$

Equation (34) is the semiclassical solution to the inverse bound-state problem. Although we never applied it to the quarkonium problem, it stimulated us to think of ways to reconstruct the interquark potential from the properties of the narrow quarkonium levels.

THE INVERSE BOUND-STATE PROBLEM

In one space dimension, binding energies and phase shifts uniquely define a symmetric potential $V(x) = V(-x)$, for which $V(\infty)$ approaches a constant

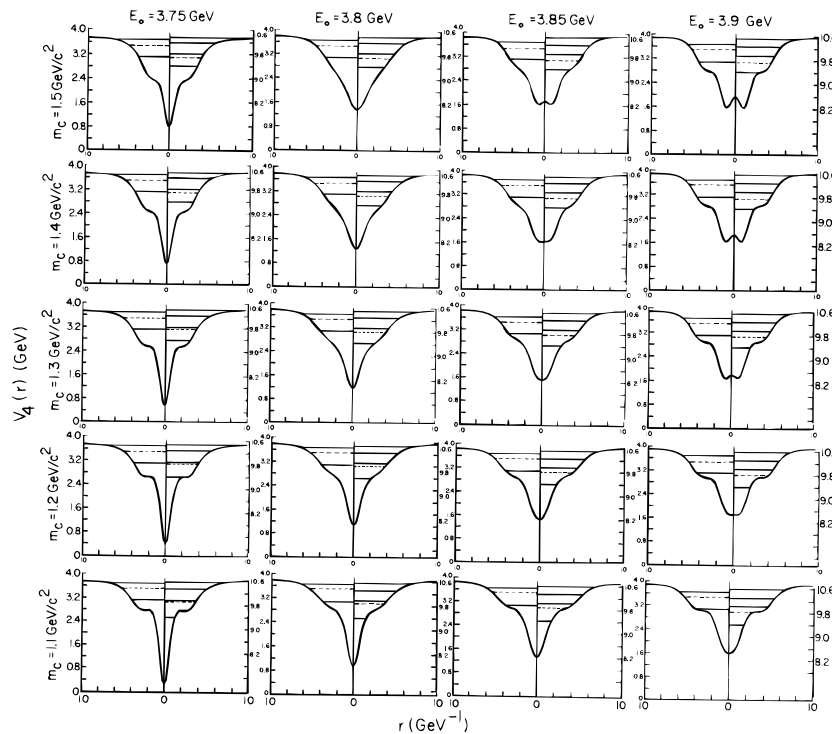


FIGURE 6. Interquark potentials reconstructed from the masses and leptonic widths of $\psi(3.095)$ and $\psi'(3.684)$. The levels of charmonium are indicated on the left-hand side of each graph. Those of the Υ family are shown on the right-hand side of each graph. The solid lines denote 3S_1 levels; dashed lines indicate the 2^3P_J levels. (From Thacker, Quigg, and Rosner [20].)

(finite) value. For a “reflectionless” potential (trivial phase shifts), $V(x)$ is an algebraic function of the binding energies [19]. For the s -wave inverse problem *in three dimensions*, the central potential is implied by binding energies and wave functions at the origin.

Thacker, Rosner, and I developed a method of successive approximation to confining potentials in terms of a sequence of reflectionless potentials that support a finite number of bound states [20]. (It is possible to prove interesting statements about the convergence of the procedure [21].) Figure 6 shows our first attempts to reconstruct potentials from what was known about the narrow $c\bar{c}$ levels, and to use those potentials to predict the properties of $b\bar{b}$ states. In time, we were able to determine potentials separately from the ψ and Υ families, and compare them. They agree remarkably well, except at the shortest distances, to which the Υ spectrum has greater sensitivity [22].

The inverse-scattering approach is free from assumptions about the short-distance and long-distance behavior of the potential. It provided additional evidence for flavor independence of the interquark potential, and gave us new information on the shape of the potential and where it is determined—for

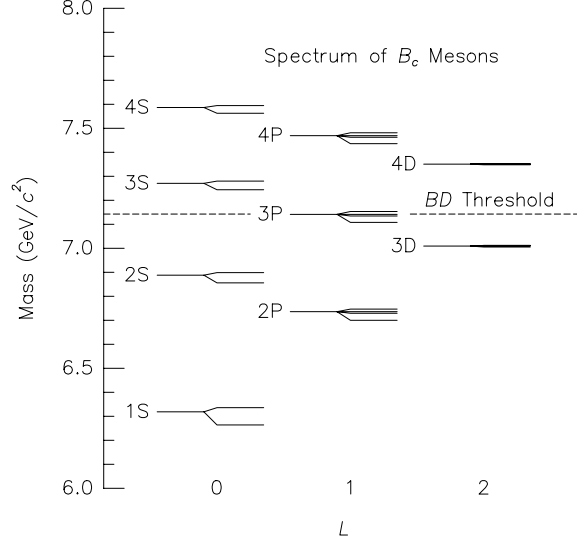


FIGURE 7. The spectrum of $b\bar{c}$ states in the Buchmüller–Tye potential (after Eichten & Quigg [24]).

$0.1 \text{ fm} \lesssim r \lesssim 1 \text{ fm}$. Some important methodological improvements have been achieved using techniques of supersymmetric quantum mechanics [23].

MESONS WITH BEAUTY AND CHARM

The next hurrah for quarkonium physics will be the experimental investigation of the B_c family of $b\bar{c}$ bound states. The B_c family is interesting as a heavy-heavy system that occupies the region of space between the J/ψ and the Υ . Since we know the heavy-quark potential in that region, we should be able to calculate the properties of the $b\bar{c}$ states reliably. Unlike the excited $c\bar{c}$ and $b\bar{b}$ states, all of the $b\bar{c}$ levels below BD threshold are stable against strong decays to light flavors ($b\bar{c} \not\rightarrow \text{gluons}$). They cascade by photonic or hadronic transitions to the B_c ground state. The interest in these states is not merely academic. They will soon be discovered and studied at the Tevatron Collider through the decays $B_c \rightarrow \psi\pi, \psi\ell\nu, \dots$

Estia Eichten and I have computed the spectrum of $b\bar{c}$ bound states in a number of interquark potentials [24]. We find that the mass of the ground state should lie in the range $M_{B_c} = 6.258 \pm 0.020 \text{ GeV}/c^2$. The low-lying levels in the Buchmüller–Tye potential [25] are shown in Figure 7. It is noteworthy that approximately 15 s, p, d -wave states lie below BD threshold. We have also computed the E1, M1, and hadronic transitions between $b\bar{c}$ levels. The transitions involving $n = 2$ and $n = 1$ states are shown in Figure 8. The narrow widths of these excited states are gathered in Table 2. We find that the deep binding of the heavy quarks has a profound influence on decay rates of the B_c . For example, we estimate that $f_{B_c} \approx 420 \text{ MeV} \gtrsim 3f_\pi$, which implies

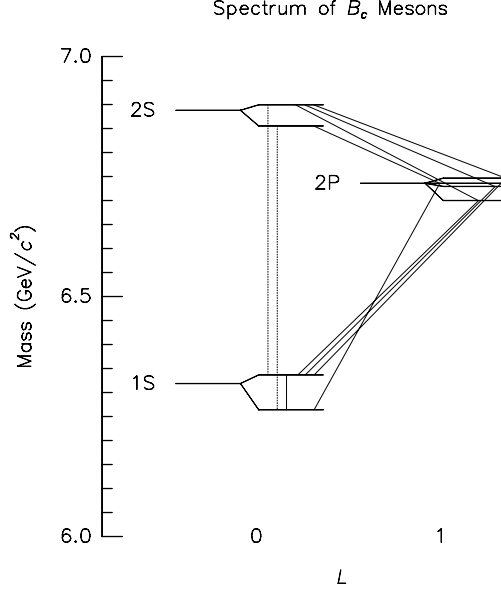


FIGURE 8. Prominent transitions in the B_c spectrum (after Eichten & Quigg [24]).

TABLE 2. Total widths of low-lying excited states of B_c .

$b\bar{c}$ State	Total Width [keV]
1^3S_1	0.135
2^1S_0	55
2^3S_1	90
2^3P_0	79
$2(1^+)$	100
$2(1^{+'})$	56
2^3P_2	113

that the purely leptonic decay $B_c \rightarrow \tau \nu_\tau$ will be unusually prominent.

In view of the CDF Collaboration's success in reconstructing the χ_c and χ_b states, I believe that a reasonable—though challenging—experimental goal will be to map the eight lowest-lying $b\bar{c}$ states ($1S$, $2S$, $2P$) through the transitions $2^3S_1 \rightarrow 1^3S_1 + \pi\pi$, $2^1S_0 \rightarrow 1^1S_0 + \pi\pi$, $B_c + 455\text{-MeV } \gamma$ s, and $(B_c^* \rightarrow B_c \gamma(72 \text{ MeV})) + 353\text{-}, 382\text{-}, 397\text{-MeV } \gamma$ s.

Phenomenological issues raised by the B_c family include the systematics of spin splittings for the unequal-mass $b\bar{c}$ system and the importance of relativistic $\mathcal{O}(\beta^2)$ corrections. As a third quarkonium system, B_c should provide a splendid test of *a priori* calculations from lattice QCD.

ENVOI

What have we learned from two decades of quarkonium spectroscopy? The first lesson, which underlies all the others, is that nonrelativistic quantum mechanics is an appropriate tool for interpreting the quarkonium spectra. Using this tool, we have been able to demonstrate by comparing the $c\bar{c}$ and $b\bar{b}$ systems that the force between quarks is flavor independent, as we expect from QCD. Moreover, we have been able to map the interaction between heavy quarks in the range $0.1 \text{ fm} \lesssim r \lesssim 1 \text{ fm}$.

The potential-model approach allows a predictive spectroscopy, including calculations of spin splittings, E1 transition rates, the characteristics of $2S \rightarrow 1S$ hadronic transitions, and the properties of wave functions at the origin, which are crucial inputs for calculations of quarkonium production in hadronic interactions.

It goes without saying that we have also learned a lot about quantum mechanics!

What can we hope to learn in the years to come? Among experimental goals, we should endeavor to complete the charmonium spectrum: refine our knowledge of the η_c and the 3P_J states, confirm the $^1P_1 (h_c)$ level, find the η'_c , and search for narrow D-states. It is also desirable to expand our knowledge of the Υ spectrum: locate the η_b and η'_b and the $1^1P_1 (h_b)$ level, and search for 1D and 2D states. I am optimistic that we shall soon find the B_c and begin to explore the $b\bar{c}$ spectrum.

On the theoretical side, we should be able to refine our understanding of relativistic effects and spin splittings. (Information from the B_c spectrum should help, in time.) It may be profitable to revisit the coupled-channel effects that influence the spectrum near and above the flavor threshold. There may be lessons of value for B -factory experiments here. And finally, we theorists are determined to solve QCD and predict the interaction between heavy quarks.

ACKNOWLEDGEMENTS

It is a pleasure to thank Dan Kaplan and our other IIT hosts for the invitation to speak and for four pleasant and stimulating days in Chicago. I am grateful to Stew Smith and the Princeton University Physics Department for generous hospitality during the spring semester of 1997. Kyoko Kunori provided valuable calligraphic assistance [1]. I am indebted to Ken Lane, Fermilab Visual Media Services, and the CERN Public Information Office for historical photographs shown in my talk.

I am happy to have this opportunity to thank my quarkonium friends and collaborators, Hank Thacker, Waikwok Kwong, Jonathan Schonfeld, Peter Moxhay, Taiji Yamanouchi, Leon Lederman, André Martin, and especially

Jon Rosner and Estia Eichten. I also want to thank absent friends Ben Lee, John Bell, and S. Chandrasekhar for many important lessons.

REFERENCES

1. T. Appelquist and H. D. Politzer, *Phys. Rev. Lett.* **34**, 43 (1975).
2. See Kurt Gottfried's excellent summary of the work of the Cornell group, *These Proceedings*.
3. E. Eichten and K. Gottfried, *Phys. Lett.* **66B**, 286 (1977).
4. C. Quigg and J. L. Rosner, "Quarkonium Level Spacings," *Phys. Lett.* **71B**, 153 (1977).
5. R. J. Perry, "A Simple Confinement Mechanism for Light-Front Quantum Chromodynamics," (electronic archive: hep-th/9411037); M. M. Brisudova, R. J. Perry, K. G. Wilson, *Phys. Rev. Lett.* **78**, 1227 (1997).
6. C. Quigg and J. L. Rosner, "Scaling the Schrödinger Equation," *Comments Nucl. Part. Phys.* **8A**, 11 (1978).
7. J. L. Rosner, C. Quigg, H. B. Thacker, "Measuring the Fifth Quark's Charge: The Role of Υ Leptonic Widths," *Phys. Lett.* **74B**, 350 (1978).
8. See the summary of Υ and Υ' properties reported in Session B1, on p. 265 of the *Proceedings of the 19th International Conference on High Energy Physics*, Tokyo, 1978, edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Physical Society of Japan, Tokyo, 1979).
9. Their work is surveyed in H. Grosse and A. Martin, "Exact Results on Potential Models for Quarkonium Systems," *Phys. Rep.* **60**, 341 (1980).
10. J. L. Rosner, "Charm and Beauty in Particle Physics," an appreciation on the occasion of André Martin's retirement, (electronic archive: hep-ph/9501291).
11. A. Martin, *Phys. Lett. B* **93**, 338 (1980); in *Heavy Flavours and High Energy Collisions in the 1-100 TeV Range*, edited by A. Ali and L. Cifarelli (Plenum Press, New York, 1989), p. 141.
12. S. Chandrasekhar, *Newton's Principia for the Common Reader* (Clarendon Press, Oxford, 1995).
13. C. Quigg and J. L. Rosner, "Quantum Mechanics with Applications to Quarkonium," *Phys. Rep.* **56**, 167 (1979).
14. A. Grant and J. L. Rosner, "Classical Orbits in Power-Law Potentials," *Am. J. Phys.* **62**, 310 (1994).
15. C. Quigg and J. L. Rosner, "Counting Narrow Levels of Quarkonium," *Phys. Lett.* **72B**, 462 (1978).
16. C. Quigg and J. L. Rosner, "Semiclassical Sum Rules," *Phys. Rev. D* **17**, 2364 (1978).
17. J. S. Bell and J. Pasupathy, *Phys. Lett.* **83B**, 389 (1979); *Z. Phys. C* **2**, 183 (1979).
18. P. Moxhay and J. L. Rosner, *J. Math. Phys.* **21**, 183 (1980).
19. For the case of confining potentials, see H. B. Thacker, C. Quigg, and

- J. L. Rosner, “Inverse Scattering Approach to Quarkonium Potentials. I: One-Dimensional Formalism and Methodology,” *Phys. Rev. D***18**, 274 (1978).
20. H. B. Thacker, C. Quigg, and J. L. Rosner, “Inverse Scattering Approach to Quarkonium Potentials. II: Applications to ψ and Υ Families,” *Phys. Rev. D***18**, 287 (1978).
 21. J. F. Schonfeld, *et al.*, “On the Convergence of Reflectionless Approximations to Confining Potentials,” *Ann. Phys. (NY)* **128**, 1 (1980).
 22. C. Quigg, H. B. Thacker, and J. L. Rosner, “Constructive Evidence for Flavor Independence of the Quark-Antiquark Potential,” *Phys. Rev. D***21**, 234 (1980); C. Quigg and J. L. Rosner, “Further Evidence for Flavor-Independence of the Quark-Antiquark Potential,” *Phys. Rev. D***23**, 2625 (1981).
 23. W. Kwong and J. L. Rosner, in the Nambu Festschrift volume, *Prog. Theor. Phys. (Suppl.)* **86**, 366 (1986); W. Kwong, H. Riggs, J. L. Rosner, and H. B. Thacker, *Phys. Rev. D***39**, 1242 (1989).
 24. E. J. Eichten and C. Quigg, *Phys. Rev. D***49**, 5845 (1994).
 25. W. Buchmüller and S.-H. H. Tye, *Phys. Rev. D***24**, 132 (1981).